This paper intends to provide possible explanations for the empirical failure of the Fisher hypothesis in terms of economic shocks by employing the quantile cointegration methodology recently proposed by Xiao (2009). Our empirical results for six OECD countries suggest that though the nominal interest rate and inflation move together in the long run, the cointegrating coefficients between the two variables display an asymmetric pattern depending on the sign and size of the shocks, in sharp contrast to the counterparts with the conventional cointegration methods. In details, in the lower quantiles, the nominal rate is low, and would rise less proportionally than the inflation, leading to the so-called Fisher effect puzzle; by contrast, in the upper quantiles where the level of the nominal rate is high, the former would adjust on a one-to-one basis to changes in the latter, and therefore, support the Fisher hypothesis. Asymmetric monetary policies may be responsible for the findings. Finally, a further checking shows that our findings are robust to the changes of econometric modeling and data frequency.

1. Introduction

The Fisher hypothesis suggests that nominal interest rates and expected rates of inflation should move together one-for-one in the long run. It is a fundamental equilibrium asset pricing condition, and constitutes a building block for derivation of the real interest rate parity and for part of macroeconomic models. According to the full Fisher effect, the real interest rate—the nominal interest rate adjusted to expected inflation—is entirely determined by the real factors such as the marginal productivity of capital and the time preference, implying monetary super-neutrality holds and no money illusion.

Though important in the theoretical perspective, unfortunately, the Fisher hypothesis is hard to justify empirically. One observation from the related studies is that only limited evidence indicates that the two variables form a long-run equilibrium relationship by using conventional cointegration methods. The other is that even though they are cointegrated, changes in inflation expectations are not fully reflected in the adjustments of nominal interest rate. The less than proportional reaction of the interest rates to changes in expected inflation is known as the Fisher effect puzzle (e.g., Christopoulos and León-Ledesma, 2007).

Since the pioneer work of Rose (1988), who fails to find strong evidence to support stationary real interest rates for many industrial countries including the US, voluminous studies have focused on this issue by either using various unit root tests to directly examine the stationarity property of the real interest rates or employing cointegration methods to see if nominal...
interest rates and inflation are cointegrated. By assuming constant dynamics, i.e. the adjustment speed towards the long-run equilibrium level is the same, for the real interest rates or for the system of nominal interest rates and inflation, most studies still cannot uncover extensive evidence in favor of the Fisher hypothesis (e.g. Rapach and Weber, 2004).²

Recently, empirical research has resorted to the nonlinear methodologies trying to fix such weakness and has unveiled some evidence supporting the Fisher effect. For example, the results in Kapetanios et al. (2003) and Koustas and Lamarche (2007) suggest that the real interest rates are mean reverting under the ESTAR and 3-regime SETAR frameworks, respectively. Researchers like Bierens (2000) and Lanne (2006) find that nonlinear common trends may exist between nominal interest rates and inflation. Christopoulos and León-Ledesma (2007) claim that the presence of nonlinearities in the long-run equilibrium relationship between nominal interest rates and inflation rates may be responsible for the failure of the Fisher hypothesis found in the previous studies. The aforementioned discussion gives us a hint that allowing different speeds of adjustment for the underlying variables, probably, is the key to empirical validity of the Fisher relationship.

Following this observation, in this paper, we employ the quantile cointegration model recently developed by Xiao (2009) in an attempt to investigate the Fisher effect in six OECD countries and to provide complementary evidence on the dynamic relationship between nominal interest rates and inflation. Compared with the methods utilized in the previous studies, this approach has several strengths. First, as noted in Xiao (2009), it permits the values of cointegrating coefficients to be influenced by the shock received in each period, and therefore, they may vary over different innovation quantiles. In addition, it may be regarded as a stochastic cointegration model which includes the conventional counterpart as a special case. Specifically, instead of focusing on the average relationship between nominal interest rates and inflation through conditional mean function, the quantile cointegration model studies their long-run equilibrium relationship in a range of quantiles of a shock. As a result, it can quantify the impact of the signs and magnitudes of a shock on the long-run vector. In other words, potential sign asymmetry—different values of the cointegrating coefficients dependent on positive or negative shocks—can be detected.

Second, it not only allows one to investigate whether or not the quantile long-run equilibrium relationship exists between the two variables, but also provides a rigorous procedure to test in which quantiles the nominal interest rate would rise by unity in response to changes in the inflation, i.e. no Fisher effect puzzle. In other words, it offers a novel way to study possible local validity of Fisher effect, in which within certain quantiles, the Fisher equation holds, but within the others, the Fisher effect puzzle exists. Moreover, based on the results from the conditional quantiles for nominal interest rates, we devise a method to identify the periods in which the Fisher hypothesis holds for a given sample. To the best of our knowledge, this is the first attempt in the literature to explain the Fisher hypothesis in this way.

Finally, instead of assuming the innovation to follow a particular distribution, the shocks analyzed are actual, whose magnitudes are determined endogenously by the data. Due to better data descriptions accommodating potential heavy-tailed behavior in the disturbance, the quantile cointegration methodology can achieve large efficiency gain compared with the conventional least squared-based counterparts as noted by Xiao (2009). Undoubtedly, this means that the empirical results obtained with this method would be more reliable.

Our empirical results suggest that the nominal interest rates and inflation rates constitute long-run equilibrium relationships in six OECD countries based on the quantile cointegrating method, in sharp contrast to the results from the Engle–Granger procedure that there is no cointegrating relationship between the two variables and that the inflation coefficient is less than unity. In addition, both the sign and size of the shocks, generally, have significant impact on the cointegrating coefficients, and therefore sign asymmetry is evident. Most importantly, strong evidence is unveiled to support local validity of the Fisher effect, in which in the upper quantiles associated with higher levels of nominal rates, nominal interest rates and inflation form a one-to-one relationship, while in the medium or lower quantiles, the former variable rises by less than unity responding to changes in the latter. These results imply that when inflation is above a certain level due to economic recoveries, monetary authorities would increase interest rate aggressively in an attempt to curb inflation, and therefore, lead to the validity of the Fisher hypothesis. However, even though inflation is low probably due to economic downturns, monetary authorities would not lower enough percentage of nominal interest rate to stimulate the economy since they are afraid of the recurrence of high inflation to undermine the economy. Such cautious reaction results in less than unity coefficients for inflation.

The remainder of the paper is organized as follows. In Section 2, we present the methodology used in this study. The empirical results are collected in Section 3. Section 4 conducts robustness analyses. Section 5 concludes the paper.

2. The Fisher effect and methodologies

2.1. The Fisher effect

According to the Fisher hypothesis, the real interest rate \( r_t \) is expressed as \( r_t = R_t - \pi_t^e \), where \( R_t \) is the nominal interest rate and \( \pi_t^e \) is the expected inflation rate. Since the expected inflation is not directly observable, as in most empirical studies like Rose (1988) and Rapach and Weber (2004), the actual inflation rate is often used as a proxy under the assumption of

² Please refer to Neely and Rapach (2008) for a detailed survey.
stationary inflation forecast errors. To be precise, under rational expectations, the expected inflation is the sum of the actual inflation \((\pi_t)\) and a stationary forecast error \((e_t)\) with zero mean, i.e. \(\pi_t^e = \pi_t + e_t\), and therefore, we have
\[
r_t = R_t - \pi_t - e_t. \tag{1}
\]

The above equation implies that under the assumption of rational expectations, the Fisher equation holds if nominal interest rate and inflation are cointegrated with cointegrating vector of \((1, -1)\) provided that they are non-stationary. In practice, this proposition is usually investigated in the following regression:
\[
R_t = \beta_0 + \beta_1 \pi_t + u_t. \tag{2}
\]

If the two variables are tested to be cointegrated with \(\beta_1 = 1\), then the full Fisher effect holds; otherwise if \(\beta_1 < 1\), the Fisher effect puzzle exists. Another way commonly seen in the literature is to directly test the stationarity of the real interest rate by using unit root tests. As we can see in Eq. (1), this approach is equivalent to testing the existence of cointegration with imposing cointegrating coefficient \(\beta_1 = 1\). In this paper, we will adopt the former strategy to examine the Fisher hypothesis.

### 2.2. Empirical methodologies

Consider the following model:
\[
y_t = \beta_1 x_t + u_t, \quad t = 1, 2, \ldots, n. \tag{3}
\]
where \(y_t = R_t - \bar{R}, x_t = \pi_t - \bar{\pi}\), with \(\bar{R}\) and \(\bar{\pi}\) denoting the long run levels of \(R_t\) and \(\pi_t\), respectively. After decomposing \(u_t\) into sum of the lead and lag terms of \(\Delta x_t\) and a pure innovation \(\varepsilon_t\) to eliminate the second-order bias originating from the correlation between \(x_t\) and \(u_t\), the original model in Eq. (3) can be rewritten as follows:
\[
y_t = \beta_1 x_t + \sum_{j=-K}^{K} \delta_j \Delta x_{t-j} + \varepsilon_t. \tag{4}
\]

Following the methodology in Xiao (2009), the \(r\)th conditional quantile of \(y_t\) takes the form:
\[
Q_{y_t}(r|X_t) = \beta_0(t) + \beta_1(t)x_t + \sum_{j=-K}^{K} \delta_j(t)\Delta x_{t-j}, \tag{5}
\]
where \(X_t\) is the information accumulated up to time \(t\), and \(\beta_1(t)\) denotes the \(r\)th conditional quantile of \(\varepsilon_t\). Note that in Eq. (5), \(\beta_1(t)\) is the cointegrating coefficient that can be affected by the shock, and therefore, can vary over the quantiles.

Estimation of the parameters in Eq. (5) involves solving the problem
\[
\text{Min } \sum \rho_\tau(y_t - \beta_0(t) - \beta_1(t)x_t - \sum_{j=-K}^{K} \delta_j(t)\Delta x_{t-j}). \tag{6}
\]
where \(\rho_\tau(u) = u(\tau - I(u < 0))\) with \(I\) representing an indicator function. After solving Eq. (6), we can test \(H_0: \beta_1(t) = 1\), i.e. the full Fisher effect, by using the Wald test as follows:
\[
W_n(\tau) = \frac{\hat{f}(F_\tau^{-1}(\tau))^2}{\hat{\omega}_{\tau}^2} (\hat{\beta}_1(t) - 1)^2 \sum (X_t - \bar{X})^2, \tag{7}
\]
where \(\hat{\beta}_1(t)\) is the solution for \(\beta_1(t)\) in Eq. (6), \(\hat{f}(F_\tau^{-1}(\tau))\) is a consistent estimator of \(f(F_\tau^{-1}(\tau))\), with \(f\) and \(F\) representing the density and distribution function of \(\varepsilon_t\) in Eq. (4), \(\hat{\omega}_{\tau}^2\) is an estimator for \(\omega_{\tau}^2\), the long-run variance of \(\psi_{\tau}(\varepsilon_{t\tau}) = \tau - I(\varepsilon_{t\tau} < 0), \) with \(\varepsilon_{t\tau} = \varepsilon_t - F_\tau^{-1}(\tau)\). Under the null hypothesis, the \(W_n(\tau)\) statistic is Chi-square distributed asymptotically with one degree of freedom.

Moreover, the quantile cointegration approach allows one to investigate whether the Fisher coefficients \((\beta_1(t))\) are constant over a range of given quantiles. Specifically, the null hypothesis of \(H_0: \beta_1(t) = \beta_1\) over \(\tau \in \Gamma\) can be tested based on the statistic of sup \(V_n(\tau)\), where \(\hat{\beta}_1\) is the least squares estimate for \(\beta_1\) in Eq. (4), and \(V_n(\tau) = n(\hat{\beta}_1(\tau) - \beta_1)\). We calculate \(V_n(\tau)\) over the set of \(\Gamma = \{0.1, 0.2, \ldots, 0.9\}\) in the following empirical study, and thus the test statistic can be constructed by taking maximum over \(\Gamma\). Unfortunately, the asymptotic distribution of the test is nonstandard and is dependent on several nuisance parameters. To make the test viable, Xiao (2009) suggests a bootstrap scheme to construct its empirical critical values.\(^5\) The number of repetitions is 1000 in the following empirical study. Additionally, based on the fluctuation of the residuals from the quantile cointegration regression, Xiao (2009) proposes a quantile cointegration test, denoted by sup \(V_n|\), to examine the null hypothesis of quantile cointegration.\(^5\) In the following section, the aforementioned estimation and inference techniques are employed to elaborate on the Fisher effect.

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\(^3\) For brevity, the computation of \(f\) and \(\hat{\omega}_{\tau}^2\) are omitted. Please refer to Xiao (2009) for details.

\(^4\) Such re-sampling procedure is omitted to conserve space. Please refer to Xiao (2009) for details.

\(^5\) This is also omitted for brevity. For further information, please refer to Xiao (2009).
3. Empirical results

3.1. Data and preliminary results

The quarterly data used in this study includes 3-month Treasury-Bill rate and the annual growth rate of CPI retrieved from International Monetary Fund’s International Financial Statistics (IFS) database. Six OECD countries are considered here, including Australia, Belgium, Canada, Sweden, the United Kingdom, and the United States. The inflation rate was appropriately aligned with the quarterly observations of the interest rates. In most cases, the data spans from 1957Q1 to 2010Q2. In Fig. 1, we plot the Treasury-Bill rates and the inflation rates for each country. As shown in Fig. 1, the two series appear to move together for each country.

Tables 1.1 and 1.2 report the first four sample moments of the inflation rates and the nominal interest rates, and their respective results of the Jarque–Bera (JB) normality tests. The inflation rate in UK has the largest long-run mean (5.812) and standard deviation (5.074). The JB test indicates that these inflation rates exhibit fat-tailed and non-normal behavior due to the small p-values. For the nominal interest rates, Australia has the largest long-run mean (8.777), while the US has the smallest counterpart (5.164). The largest and smallest sample standard deviations are 3.856 and 2.843 for Australia and the US, respectively. Again, all the interest rates display heavy tail and non-normality since the JB test rejects the null hypothesis of normality at the 5% significance level, except for Belgium. The marked evidence of non-normality in the data is supportive of our usage of the quantile cointegration approach in this study as claimed by Xiao (2009).

Though our focus is on the results of quantile cointegration, the counterparts from the conventional testing procedure of Engle and Granger (1987) are also presented for comparison. Before resorting to the cointegration method, we must ensure the interest rates and inflation rates follow an integrated process. They are examined with the DF-GLS test, in which the lag length is selected by the MAIC proposed by Ng and Perron (2001) to achieve good size and power properties.

The results, collected in Table 2.1, indicate that the unit-root null cannot be rejected by the DF-GLS test for the variables across all countries, since all the test values are larger than −1.98, the 5% critical value. The MAIC chose larger lags in most countries to account for the possibility of a negative MA root in the inflation series, but shorter lags for the nominal interest rates. These results are consistent with the findings in the literature such as Rapach and Weber (2004), and Lee and Tsong (2011). As commonly mentioned in the literature (e.g., Perron, 1989) that the unit root tests are biased towards accepting the null hypothesis of unit root in the presence of a structural break. Hence, we also perform the unit root test of Perron and Rodriguez (2003) on the two variables. The test, denoted by the DF-GLS-B test, investigates unit roots using GLS detrending, allowing for a break point at an unknown date. As shown in Table 2.2, even after taking into account the presence of a mean break, we still fail to reject the unit-root null in the both series, in line with the counterparts in Table 2.1.

Next, the two-step cointegration testing procedure of Engle and Granger (1987) is applied to the data. The results are collected in Table 3. The evidence in Table 3 indicates that the long-run equilibrium relationship between the nominal interest rate and the inflation rate does not exist for all countries since the null hypothesis of no cointegration fails to be rejected by the ADF test. This is in conformity with the results in Rapach and Weber (2004). Moreover, the Fisher coefficients are smaller than one, ranging from 0.4 (Australia) to 0.846 (Canada). Consequently, the nominal rates would unanimously rise by less than unity in response to changes in the inflation rate, resulting in the Fisher effect puzzle. This is in line with the results found by Christopoulos and León-Ledesma (2007) with the linear cointegration method.

3.2. Quantile cointegration results

Table 4 reports the results of quantile cointegration for a range of quantiles, including the estimated values of constant term (β0(τ)), cointegrating coefficient (β1(τ)), the Wald, sup |Vn| and sup |Yn| tests, along with their respective p-values. Notice that the p-value for β0(τ) is investigating the null of zero with student-t test, while the counterpart for the Wald test is testing the null of the full Fisher effect (β1(τ) = 1), as we mentioned earlier. In addition, the lag order K in Eq. (4) is set at 2.

We first focus on the sup |Vn| test, which gives an overall viewpoint of the long-run relationship between the nominal interest rates and inflation rates in question. The results of the sup |Yn| test in Table 4 offer marked evidence supporting that the two variables exhibit long run equilibrium relationship in a quantile sense for all countries, since the null hypothesis of quantile cointegration cannot be rejected due to the large p-values. These are in sharp contrast to the counterparts in Table 3 where the linear cointegrating equilibrium is rejected by using the Engle–Granger method. Moreover, the quantile-varying cointegrating coefficients are further confirmed strongly by the sup |Vn| test due to the extremely small p-values, implying that the cointegration model with constant coefficients may be subject to misspecification.

For further investigation, we turn our attention to the long-run relationship between the two variables in each specific quantile. The first observation is that the estimated values for intercept β0(τ) and cointegrating coefficient β1(τ) are different across various quantiles. As mentioned before, β0(τ) represents the magnitude of the observed shock within the τth quantile that hits the nominal interest rate. The estimated value of β0(τ) with positive (negative) sign implies positive (negative)
As we can see in Table 5, $\beta_1$ and $\beta_2$ are strictly positive and significantly different from zero, consistent with the economic intuition. More importantly, the null hypothesis $H^0_{b1}$: $\beta_1 = 1$ is rejected at the 5% level for all countries since the values of Wald test are quite large and the corresponding p-values are close to zero, implying that the Fisher effect puzzle does exist for these observations of recession periods. Meanwhile, the null of $H^0_{b2}$: $\beta_1 + \beta_2 = 1$ cannot be rejected for all case except for the US, which is rejected only at the 10% level. These results reaffirm the validity and usefulness of the quantile cointegration methodology used in our empirical study.

In the second one, we carry out all the quantile estimation and testing procedures using monthly data, obtained from the IFS database. Note that Australia is omitted due to CPI data is unavailable.

As shown in Table 6, all in all, the main findings in Section 3.2 do not change. They are summarized as follows. First, the shocks are of negative sign in the low quantiles, while they are of positive sign in the upper quantiles. The magnitudes of shock related to nominal interest rate increase with quantiles. Second, the cointegrating coefficients between the two variables display an asymmetric pattern depending on the sign and size of the shocks. To be specific, the Fisher coefficients are large and close to unity in the upper quantiles, compared with the counterparts in the lower quantiles that they are smaller than unity. Further, the Wald test cannot reject the null of unity Fisher coefficient in the upper quantiles, and can reject the same null in the small quantiles. These implies that the Fisher hypothesis holds when nominal interest rates are hit by positive and large shocks, and fails to hold otherwise. Finally, a long-run equilibrium relationship exists between nominal interest rates and inflation in a quantile sense.

5. Conclusions

In this paper, we re-investigate the long-run relationship between nominal interest rates and inflation by using quantile cointegration estimation and inference. Instead of relying on a single measure of conditional central tendency like the conventional cointegration methods, these approaches enable us to explore their relationship with quantile-varying cointegrating coefficients. In addition, in comparison with the conventional counterparts, the quantile approaches makes no assumption about the distribution of the regression error, and can accommodate its potential fat-tailed characteristic. Moreover, the quantile approaches may be viewed as a stochastic cointegration model, which includes the conventional methods as a special case. Due to such better data descriptions, the quantile methods can lead to substantial efficiency gains and to more reliable empirical results.

Most importantly, they allow us to analyze possible asymmetry in Fisher relationship, in which the long-run coefficient of inflation depends on the signs and magnitudes of the shock. Consequently, they provide important insights on the long-run equilibrium relationship between the two variables, and can identify whether or not the Fisher effect puzzle exists in certain quantiles in a rigorous way.

Our empirical results suggest that the Fisher hypothesis holds in the six OECD countries in a quantile sense, in sharp contrast to the counterparts from the conventional cointegration method. We also find that in the upper quantiles, the Fisher coefficient is not significantly different from unity, meaning that the nominal interest rate responds one-to-one to changes in inflation, and therefore support the Fisher hypothesis. However, in the lower quantiles, the nominal interest rate would rise by less than unity in response to changes in the inflation rate. The evidence of partial adjustment lends strong support to the existence of Fisher effect puzzle.

The stances policymakers take to determine the nominal interest rate as a tool for stabilizing price level could be a reasonable explanation. Given that the monetary authorities are capable of determining the short-term rate to stabilize inflation, our results imply that when inflation is high, they would increase nominal interest rate more aggressively to bring down the inflation and try to prevent high inflation from hurting the economy, therefore, resulting in a one-to-one relationship between the two variables. However, when the inflation is low due to economic downturns, they just lower the nominal interest rate less proportionally than the decrease in inflation because of being afraid of the resurgence of high inflation. Such partial adjustments of nominal interest rate result in the Fisher effect puzzle.

References


