A three-dimensional heat sink module design problem with experimental verification

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A B S T R A C T

A three-dimensional heat sink module design problem is examined in this work to estimate the optimum design variables using the Levenberg–Marquardt Method (LMM) and a general purpose commercial code CFD-ACE+. Three different types of heat sinks are designed based on the original fin arrays with a fixed volume. The objective of this study is to minimize the maximum temperature in the fin array and to determine the best shape of heat sink. Results obtained by using the LMM to solve this 3-D heat sink module design problem are firstly justified based on the numerical experiments and it is concluded that for all three cases, the optimum fin height H tends to become higher and optimum fin thickness W tends to become thinner than the original fin array, as a result both the fin pitch D and heat sink base thickness U are increased. The maximum temperature for the designed fin array can be decreased drastically by utilizing the present fin design algorithm. Finally, temperature distributions for the optimal heat sink modules are measured using thermal camera and compared with the numerical solutions to justify the validity of the present design.

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1. Introduction

Nowadays, the tendency to design electronic products becomes lighter, thinner, shorter, and smaller. Due to the fact that shrinking in the dimension of these electronic products will result in drastic increase in the heat generation rate when comparing with previous products. For this reason, an efficient cooling system to remove the high heat generation, and consequently maintain the stability and reliability of the products, have received much attention.

The heat sink module is the most common heat exchanger for CPUs and has been extensively used in order to provide cooling function for electronic components. The conventional heat sink module utilized the forced convection cooling technique; dissipate heat from CPUs to the ambient air. The combination of the fan and heat sink design usually involved in this forced convection cooling technique.

The forced convection cooling technique becomes one of the most commonly used devices to cool CPUs since it has the advantageous of simple maintenance process, more reliability and lower manufacturing cost. It has been seen by many researchers that a heat sink with good geometrical design will provide better cooling performance and higher efficiency. It implies that the optimization process must be an effective tool for the heat sink design problem.

If an efficient heat sink design algorithm is provided, it will greatly improve the reliability and prolong the life span of the CPUs. Many investigations of the optimum design parameters and the selection of heat sink module have been proposed in order to offer a high-performance heat removal characteristic. For instance, Kraus and Bar-Cohen [1] presented the fundamental theories for heat transfer and hydrodynamics characteristics of heat sinks. Shih and Liu [2] and Furukawa and Yang [3] presented an approach to design the plate-fin heat sinks by minimizing the entropy generation rate in order to reach the most efficient heat transfer. Leon et al. [4] and Small et al. [5] used computational fluid dynamics (CFD) to study flow and heat transfer behaviors for staggered heat sinks in detail. Iyengar and Bar-Cohen [6] utilized the least-energy optimization algorithm to design the plate fin heat sinks in the forced convection problem. Yang and Peng [7,8] investigated numerically the thermal performances of the heat sink with un-uniform fin width and fin height designs with an impingement cooling. Zhou et al. [9] considered a multi-parameter constrained optimization procedure to design the plate finned heat sinks by minimizing their rates of entropy generation. Park and Moon [10] utilized the progressive quadratic response surface model to estimate the optimum fin design variables for a plate-fin type heat sink. Srisomporn and Bureerat [11] considered a geometrical design problem for the plate-fin heat sinks by using hybridization of the multiobjective evolutionary algorithms (MOEAs) and a response surface method (RSM). Shan et al. [12] established the
A direct link between the pressure drop of heat sinks and system operating curve for the selected fan to optimize a parallel plate impingement heat sink. Park et al. [13] applied the numerical optimization to determine the shape of pin-fins for a heat sink to improve the cooling efficiency. Sahin et al. [14] used the Taguchi experimental design method to examine the effects of design parameters on the heat transfer and pressure drop characteristic of a heat exchanger.

From references mentioned above, the optimum design problems for an efficient heat sink module become a primary challenge in the electronic industry. In order to obtain an optimum design for heat sink modules, the proper types for heat sink modules and a suitable optimization algorithm should be chosen before proceeding to the design problem. Among numerous existing designs of heat sink modules, the design of heat sink with the plate fin array is widely utilized in the cooling enhancement of current electronic equipment. Therefore this type of heat sink module with modifications will be considered in this work. Besides, the present study will also focus on the thermal performance of the fan–sink assembly subjected to forced air cooling.

The Levenberg–Marquardt Method (LMM) [15] has proven to be a powerful algorithm in inverse design calculation for engineering applications. This inverse design method had been applied to predict the form of a ship’s hull in accordance with the desired hull pressure distribution by Huang et al. [16]. Subsequently, Chen and Huang [17] applied it to predict an unknown hull form based on the preferable wake distribution in the propeller disk plane. Chen et al. [18] further applied it to the aspect of optimal design for a bulbous bow. Huang and Lin [19] applied LMM in the theoretical and experimental Studies to estimate the optimum shape for gas channel for a serpentine PEMFC. The LMM will be adopted in the present study as an optimization algorithm.

This work addresses the development of an efficient method for parameter estimation in estimating the design variables for heat sink modules that satisfies the constraint of minimizing the maximum surface temperature. However, without experimental verification it is difficult to show that the present design algorithm can be utilized in reality. For this reason in the present study the estimated optimal heat sinks will be fabricated and they will be used in experiment to measure the temperatures by using infrared thermal scanner. Finally these temperatures will be compared with the calculated temperatures to show the accuracy of our computations.

2. The direct problem

The following three-dimensional heat sink module is considered to illustrate the methodology for developing expressions for use in determining the design variables for heat sink module in the present inverse design problem by using LMM and CFD-ACE+ [20].

It is assumed that Ω represents the domain of computation and \( \Omega = (\Omega_1 \cup \Omega_2) \), where \( \Omega_1 \) indicates the domain of fin array and \( \Omega_2 \) represents the air flow region. The boundary conditions on all the outer boundary surfaces are subjected to the Robin boundary conditions with heat transfer coefficient \( h \) and ambient temperature \( T_1 \). A heat flux \( q \) is imposed at the heating surface \( S_h \) while the rest of bottom surface \( S_b \) of fin array remains insulated.

Fig. 1(a) shows the geometry of the computational domain of heat sink module and Fig. 1(b) indicates the bottom and heating surfaces of the fin array.

The mathematical formulation of this 3-D heat conduction problem for the fin domain \( \Omega_1 \) is given by:

\[
\nabla \cdot (k \nabla T) = q
\]

**Nomenclature**

- \( B_j \): design variables
- \( D \): fin pitch (calculated variable)
- \( D_1 \): the width of row passage (calculated variable)
- \( H \): fin height (design variable)
- \( J \): functional defined by Eq. (10)
- \( k \): thermal conductivity of fin
- \( L_1, L_2 \): width and depth of the computational domain
- \( N \): number of fin plate in each row
- \( q \): applied heat flux
- \( S_b \): bottom surface
- \( S_h \): heating surface
- \( T_f \): calculated fin temperature
- \( T_\infty \): ambient temperature

**Greek symbols**

- \( \Psi \): Jacobian matrix defined by Eq. (16)
- \( \Omega \): total computational domain
- \( \varepsilon \): convergence criterion
- \( \mu^n \): damping parameter

**Fig. 1.** The (a) geometry of the computational domain of heat sink module and (b) bottom and heating surfaces of the fin array.
\[ k(\Omega_1) \left[ \frac{\partial^2 T_1(\Omega_1)}{\partial x^2} + \frac{\partial^2 T_1(\Omega_1)}{\partial y^2} + \frac{\partial^2 T_1(\Omega_1)}{\partial z^2} \right] = 0; \quad \text{in} \quad \Omega_1 \quad (1a) \]

\[ \frac{k}{\partial z} q = q; \quad \text{on the heating surface} \quad S_h \quad (1b) \]

\[ \frac{\partial T_1}{\partial z} = 0; \quad \text{on the surface} \quad (S_h - S_b) \quad (1c) \]

where \( T_1 \) indicates the fin temperature distribution and \( k \) is the thermal conductivity of fin.

For the air flow region, \( \Omega_2 \), a fan is located right at the top surface of the heat sink module to drive the air and the flow is assumed to be a three-dimensional steady and incompressible flow, in addition, the thermophysical properties of the fluid are assumed to be constant. Both the buoyancy and radiation heat transfer effects are neglected. The three-dimensional governing equations of mass, momentum, and energy in the steady turbulent main flow using the standard \( k-e \) model, turbulent kinetic energy and turbulent energy dissipation rate are shown in Eqs. (2)–(6), respectively [7,8]:

\[ \frac{\partial \rho u_i}{\partial x_i} = 0 \quad (2) \]

\[ \rho u_i \frac{\partial u_i}{\partial x_j} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ \mu_u \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] \quad (3) \]

\[ \rho u_i \frac{\partial T_s}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \frac{\mu_u}{\sigma_t} + \frac{\mu_t}{\sigma_e} \right) \frac{\partial T_s}{\partial x_j} \right] \quad (4) \]

\[ \rho u_i \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \frac{\mu}{\sigma_k} \frac{\partial k}{\partial x_j} \right) + \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} \right) - \rho e \quad (5) \]

\[ \rho u_i \frac{\partial e}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \left( \frac{\mu}{\sigma_e} \frac{\partial e}{\partial x_j} \right) + C_1 \frac{e}{k} \left( \frac{\partial k}{\partial x_j} + \frac{\partial u_i}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} \right) - C_2 \rho \frac{e^2}{k} \quad (6) \]

Since the Navier–Stokes equations are solved inside the domain, no-slip boundary condition is applied to all the walls in the domain. Therefore, at all of the surfaces \( u_i = 0 \).

The solution for the above 3-D fluid-heat conjugate problem in the irregular solid and air flow domains (\( \Omega_1 \cup \Omega_2 \)) is solved using CFD-ACE+. The direct problem considered here is concerned with the determination of the velocity and temperature distributions for the heat sink when all the boundary conditions and heat flux on \( S_h \) are known.

3. The heat sink design problem

Three commonly seen heat sink modules are used in the present study, they are (1) two-row plate fin type (Type A), (2) three-row plate fin type (Type B) and (3) four-row plate fin type (Type C), and are illustrated in Fig. 2(a)–(c), respectively. The fin array volume \( V \) can be calculated by using the following equations:

\[ V = \left\{ (W \times H \times W_1 \times N \times 2) + (L_1 \times L_2 \times U) \right\} \text{mm}^3; \quad \text{for Type A heat sink} \quad (7) \]

\[ V = \left\{ (W \times H \times W_1 \times N \times 3) + (L_1 \times L_2 \times U) \right\} \text{mm}^3; \quad \text{for Type B heat sink} \quad (8) \]

\[ V = \left\{ (W \times H \times W_1 \times N \times 4) + (L_1 \times L_2 \times U) \right\} \text{mm}^3; \quad \text{for Type C heat sink} \quad (9) \]

where \( N \) indicates number of fin plate in each row, \( L_1 \) and \( L_2 \) are the width and depth of heat sink, respectively; \( W \) and \( H \) are the fin thickness and fin height, respectively, \( W_1 \) indicates the fin width.

\[ D \text{ is the fin pitch and } D_1 \text{ represents the width of row passage. Here } H, W \text{ and } W_1 \text{ are the design variables for the heat sink modules considered here. Heat sink base thickness } U \text{ can be determined by } V \text{ and design variables while } D \text{ and } D_1 \text{ can be calculated by using } W \text{ and } W_1. \text{ Here } U, D \text{ and } D_1 \text{ represent the calculated variables. When } V, H, W \text{ and } W_1 \text{ are given, the shapes for heat sinks for Type A, B and C heat sinks can be constructed.} \]

For the heat sink design problem, a fixed fin array volume is given while the design variables \( H, W \) and \( W_1 \) are regarded as being unknown; in addition, the highest temperature on heating surface \( S_h \) of heat sink are required to become as low as possible to increase the efficiency of heat sinks. According to the heating condition and geometry of the heat sink considered here, the position of the highest temperature is located at the center of heating surface \( S_h \).

Let the desired temperatures located at the center of \( S_h \) be denoted by \( Y \), the inverse design problem can then be stated as follows: utilizing the above mentioned desired temperature \( Y \), design the optimal shapes for Type A, B and C heat sinks.

The solution of the present heat sink design problem is to be obtained in such a way that the following functional is minimized:

\[ f[T_{\text{center}}(B_j) - Y]^2 = A^T A; \quad j = 1 \text{ to } P \quad (10) \]

Here \( T_{\text{center}} \) represents the estimated or computed temperatures at the center of \( S_h \) and it is the maximum temperature in the fin array. This quantity is determined from the solution of the direct problem.
given previously by using an original fin shape. \( B \) indicates the design variable and \( P \) represents the total number of design variable. For all types of heat sinks considered here, we have \( B_j = \{ H, W, W_1 \} \) and \( P = 3 \).

4. The Levenberg–Marquardt Method (IMM) for minimization

Eq. (10) is minimized with respect to the estimated parameters \( B_j \) to obtain:

\[
\frac{\partial f_i (B_j)}{\partial B_j} = \left[ \frac{\partial T_{\text{center}}}{\partial B_j} \right] [T_{\text{center}} - Y] = 0; \quad j = 1 \text{ to } P \tag{11}
\]

Eq. (11) is linearized by expanding \( T_{\text{center}}(B_j) \) in Taylor series and retaining the first order terms. Then a damping parameter \( \mu^n \) is added to the resulting expression to improve convergence, leading to the Levenberg–Marquardt Method [15], given by:

\[
(F + \mu^n I) \Delta B = D \tag{12}
\]

\[
F = \Psi^T \Psi \tag{13}
\]

\[
D = \Psi^T A \tag{14}
\]

\[
\Delta B = B^{n+1} - B^n \tag{15}
\]

Here, the superscripts \( n \) and \( T \) represent the iteration index and transpose matrix, respectively, \( I \) is the identity matrix, and \( \Psi \) denotes the Jacobian matrix, defined as:

\[
\Psi = \frac{\partial T_{\text{center}}}{\partial B^T} \tag{16}
\]

The Jacobian matrix defined by Eq. (16) is determined by perturbing the unknown parameters \( B_j \) one at a time and computing the resulting change in temperatures on \( S_b \) from the solution of the direct problem, Eqs. (1)–(6).

Eq. (12) is now written in a form suitable for iterative calculation as:

\[
B^{n+1} = B^n + (\Psi^T \Psi + \mu^n I)^{-1} \Psi^T (T_{\text{center}} - Y) \tag{17}
\]

The algorithm for choosing this damping value \( \mu^n \) is described in detail by Marquardt [15], so it is not repeated here.

The bridge between CFD-ACE+ and LMM is the INPUT/OUTPUT files. These files should be arranged such that their format can be recognized by CFD-ACE+ and LMM. A sequence of forward problems is solved by CFD-ACE+ in an effort to update the design variables for heat sink by minimizing a residual measuring the difference between estimated and desired temperatures located on \( S_b \) under the present algorithm.

5. Computational procedure

The iterative computational procedure for the solution of this heat sink design problem using the Levenberg–Marquardt Method can be summarized as follows:

Step 1. Choose the original design variables for \( B \) at the zeroth iteration to start the computations.
Step 2. Solve the direct problem given by Eqs. (1)–(6) to obtain computed temperatures \( T_{\text{center}} \) on \( S_b \).
Step 3. Construct the Jacobian matrix in accordance with Eq. (16).
Step 4. Update \( B \) from Eq. (17).
Step 5. Check the stopping criterion \( \varepsilon \); if not satisfied go to Step 2 and iterate.

Fig. 3. (a) The schematic diagram of the experimental apparatus for this work and (b) the measured positions of the fins.
calculated temperatures for Fig. 12(a) are 316.25 K and 316.82 K, respectively, which implies only 0.181% error, the maximum error occurred at the 18th fin and the error is calculated as 0.61%. For Fig. 12(b), the averaged measured and calculated temperatures are obtained as 318.36 K and 318.85 K, respectively, which implies only 0.154% error. The maximum error occurred at the 24th fin and the error is calculated as 0.43%.

For Type C fin, the comparisons of measured and calculated temperatures at higher and lower measured positions are shown in Fig. 14(a) and (b), respectively. The averaged measured and calculated temperatures are obtained as 318.36 K and 318.85 K, respectively, which implies only 0.154% error. The maximum error occurred at the 24th fin and the error is calculated as 0.43%.

Based on the above results it can be concluded that the accuracy of the numerical solutions are guaranteed and this makes the optimal design of fin array in this work valid in reality.

8. Conclusions

The Levenberg–Marquardt Method (LMM) combined with CFD-ACE+ code was successfully applied for the solution of the three-dimensional inverse design problem to estimate the optimal design variables for heat sink modules. Three types of heat sinks were considered in the optimal design process and the objective of minimizing the maximum temperature in the fin array can always be achieved. Results based on the numerical experiments show that the optimum fin height $H$ and fin thickness $W$ tend to become higher thinner than the original fin array, respectively, as a result both the fin pitch $D$ and heat sink base thickness $U$ are both increased. The temperature distributions for optimum heat sink modules are measured using thermal camera and compared with the numerical solutions. Results show that the measured temperatures match quite well with the calculated temperatures and this is a good reference to justify the validity of the present design algorithm.

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