A new LMI condition for robust stability of discrete-time uncertain systems

Shih-Wei Kau\textsuperscript{a}, Yung-Sheng Liu\textsuperscript{a}, Lin Hong\textsuperscript{a}, Ching-Hsiang Lee\textsuperscript{a}, Chun-Hsiung Fang\textsuperscript{a,∗}, Li Lee\textsuperscript{b}

\textsuperscript{a}Department of Electrical Engineering, National Kaohsiung, University of Applied Sciences, 415 Chien-Kung Road, Kaohsiung 807, Taiwan
\textsuperscript{b}Department of Electrical Engineering, National Sun Yat-Sen University, Kaohsiung 804, Taiwan

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Abstract
In this paper, a new robust stability condition is derived for uncertain discrete-time linear systems with convex polytopic uncertainties. The condition is expressed in terms of a set of linear matrix inequalities (LMIs) involving only the vertices of the polytope domain. It enables us to determine robust stability of uncertain systems easily by solving some LMIs. A rigorous proof is given to show that an interesting result appeared recently is a special case of the proposed condition. Numerical examples also demonstrate the merit of the present condition in the aspect of conservativeness over other results in the literature.

Keywords: Robust stability; Discrete-time systems; Linear matrix inequality; Polytopic uncertainty; Parameter-dependent Lyapunov matrix

1. Introduction

Robust stability (RS) is an important issue for uncertain systems [2,4]. In the application of Lyapunov function to robust control problems, quadratic stability (QS) has played a central role. During the last decades, the QS concept has been widely used for robust analysis and control for uncertain linear systems[1,3,5,6]. By QS, the stability of a polytope of matrices can be attested by means of a feasibility test of a set of LMIs [6] involving only the vertices of the uncertainty domain.

However, the QS can lead to very conservative results in some cases. Recently, different techniques based on parameter dependent [7,8,10–15,17] or piecewise Lyapunov functions [16,18] have appeared in order to provide less conservative results. Among the above literature, some focus on the continuous-time cases [10–12,17] and others on...
the discrete-time systems [7,8,15]. In [8], sufficient LMI conditions for robust stability of uncertain discrete-times are given. The generalization of these conditions to robust D-stability has been published in [14]. The key idea used in the above papers is to introduce new variables and enlarge the dimension of the LMIs to obtain sufficient conditions for the existence of a parameter-dependent Lyapunov function. Using the same idea, a nice result has been derived in [7], where sufficient conditions for robust stability of uncertain discrete-time systems in convex bounded domains are given in terms of a set of LMIs. Such results are called extended stability (ES) conditions [9]. Although all results mentioned above improve the estimate of robust stability domain, some conservatism still remains since common matrix variables are required to satisfy the whole sets of LMIs.

Recently, an interesting RS condition that is less conservative than earlier results in most cases has appeared for uncertain discrete-time systems [15], where no common matrix variables are required but may need a larger number of LMIs. The authors show that the QS is actually a special case of their results. The merit of [15] in the aspect of conservativeness over the methods of [7,8,14] have been detailed in [9]. More recently, combining the ideas in [15,14], an improved condition for robust D-stability has been derived in [13].

In this paper, a new LMI-based robust stability condition for uncertain linear discrete-time systems is proposed. The condition provides a simple way to check robust stability. Only the feasibility of a set of LMIs involving the vertices of the uncertain domain needs to be determined. If feasible, the solutions can be convexly combined to form a parameter-dependent Lyapunov matrix which assesses the stability of any dynamic system inside the uncertainty domain. It can be shown that the RS condition [15] is a special case of the present result. An exhaustive numerical comparison among QS condition [3,5], ES condition (based on extra common variables and on an augmented LMI description [7,8,14]), RS condition [15], improved robust stability (IRS) condition [13], and the proposed new robust stability (NRS) condition is given to demonstrate the superiority of the present result.

The notation used in this paper is fairly standard. $X < 0$ (or $X \leq 0$) means that the matrix $X$ is symmetric and negative definite (or symmetric and negative semidefinite). $X^T$ denotes the transpose of $X$. The symbol $I$ represents the identity matrix with appropriate dimension.

The problem formulation is given in Section 2, where some related results published recently and the main result are also presented. The comparison of conservativeness between the NRS condition and other existing ones is made in Section 3. Section 4 presents numerical examples for demonstrating the merit of the present condition. The conclusion is located in Section 5.

### 2. An NRS condition

Let the linear discrete-time uncertain system be

$$x(k+1) = A(\delta)x(k), \quad (1)$$

where $x \in \mathbb{R}^n$ and $A(\delta) \in \mathbb{R}^{n \times n}$. The state matrix $A(\delta)$ is not precisely known but belongs to a polytopic uncertainty domain $\mathcal{A}$ defined by

$$\mathcal{A} = \left\{ A(\delta) : A(\delta) = \sum_{i=1}^{N} \delta_i A_i, \sum_{i=1}^{N} \delta_i = 1, \delta_i \geq 0 \right\}. \quad (2)$$

In (2), $A_i \in \mathbb{R}^{n \times n}$, $i = 1, 2, \ldots, N$, represent the polytope vertices and are given in advance. The robust stability problem is to determine if system (1) is stable for all $A(\delta) \in \mathcal{A}$. If yes, system (1) is termed to be robustly stable. The stability analysis of (1) with $A(\delta) \in \mathcal{A}$ can also be done with a Lyapunov function depending on the parameter $\delta$, i.e., $A(\delta)$ is asymptotically stable if and only if there exists a parameter-dependent Lyapunov matrix $P(\delta) = P^T(\delta) > 0$ such that

$$A^T(\delta)P(\delta)A(\delta) - P(\delta) < 0. \quad (3)$$
Table 1
Number of stable polytopes identified by QS, ES, RS, IRS and NRS tests for $2 \leq n \leq 5$ and $2 \leq N \leq 5$

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and

$$A_1 = \begin{bmatrix} -1.0705 & -0.8665 \\ 0.1387 & -0.1204 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.0036 & 0.6150 \\ -1.3801 & -0.5264 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0.7621 & -1.4478 \\ 1.2637 & -1.2894 \end{bmatrix}$$

which define the three stable polytopes. The three polytopes are extracted from the simulated data used in Table 1. All above uncertain systems have not been identified as stable by any of the QS, ES, RS, and IRS conditions. However, the stability can be verified using Theorem 1.

5. Conclusions

In this paper, an NRS condition is proposed for determining robust stability of linear discrete-time systems with convex polytopic uncertainties. The condition is represented in LMIs and involves only the vertices of the polytope domain. Numerical examples demonstrate that the proposed result is the least conservative in all conditions available in the literature. Although more computational effort may be needed in the condition proposed, it provides robust stability evaluation that is much more close to necessity. Application of the present ideas and results to synthesize a static output feedback controller for uncertain discrete-time systems is possible and is currently under study.

References


