Model reference adaptive control for a piezo-positioning system

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1 Introduction

Along with the fast growing of semiconductor and precision-manufacturing industry, high-precision and high performance devices are becoming more necessary. Piezoelectric (PZT) actuators are popularly applied as actuators in precision positioning systems due to its advantages of infinite displacement resolution, high speed, high bandwidth, high electrical–mechanical transformation efficiency, and little heat generation. In recent years, PZT actuators have been used in many applications that require precise positioning. These application examples include optical fiber alignment, mask alignment, high-precision machining, scanning tunneling microscopy, hard disk drive, and diamond turning machines [1–4]. Since the materials of PZT actuators are generally ferroelectric, non-linear hysteresis behaviour in response to an applied electric field is always present. It was shown that the maximum tracking error caused by the nonlinear hysteresis phenomena can be as much as 10–15% of the travelling path if the PZT actuators are operated in the open voltage driving loop system [2,5]. To improve the linearity of PZT actuator, the approach using a capacitor to compensate the hysteresis and creep phenomena was verified as effective [6]. Although an electric charge control approach can reduce the hysteresis effect [7,8], this approach requires the use of a specially designed charge drive amplifier and cause a reduction in the sensitivity of the displacement. Lately, much work has been done on feedback or feedforward–feedback control strategies for hysteresis compensation. The feedback control techniques usually did not use a precise hysteresis model, but the feedforward–feedback control scheme is designed to utilize the developed hysteresis models such as Preisach model, Maxwell model, Bouc–Wen model, polynomial model, etc. Okazaki [9] used a notch filter and a state feedback controller with a state observer. In this control design, the unmodelled phase lag resulting from neglecting the hysteresis effect in the PZT actuator control system will cause instability in a closed-loop system if sufficient phase margin is not provided. Rasmussen et al. [5] proposed a repetitive control for a piezo tool system. Since the hysteresis is not considered in the control scheme, the repetitive control with fixed control parameters may not work if there are changes in the magnitude of the reference signal or external disturbances. Jung and Kim [10] proposed a feedforward model reference control method to improve the scanning accuracy of PZT actuators in a scanning tunneling microscope. Their hysteresis model has either a local memory or symmetrical behaviour that cannot exactly reflect the hysteresis behaviour. Ge and Jouaneh [11] proposed a control technique incorporating the inverse linearized Preisach model in a feedforward loop and a PID feedback controller to improve the tracking accuracy of a PZT actuator. Croft and Devasia [12] applied an inverse polynomial model in the feedback loop to cancel the hysteresis and designed a PD feedback controller to achieve the tracking task. Choi et al. [13] presented a PID control augmented with feedback linearization loop in which the feedback linearization loop used a plant model drawn from the Maxwell slip model. It indicated that the tracking performance can be further improved by adding repetitive controller when the PZT actuator is subject to the periodic reference input signal. Tsai
and Chen [14] introduced an approximate model with variable gains and variable time-delay to represent the model of the PZT actuator with hysteresis phenomena. According to the proposed approximate model, a Smith predictor-based robust $H_{\infty}$ controller is developed to achieve high-precision tracking control of a piezoactuator. Abidi and Šabanovic [15] constructed a disturbance observer based on sliding-mode control framework to attenuate the effect of piezostage hysteresis and then to achieve high accuracy in the actuator trajectory tracking. Huang et al. [16] developed an equivalent model including the hysteresis friction force dynamics to describe the motion dynamics of the piezo-positioning stage and further derived a sliding-mode controller to improve system transient performance.

This paper presents a model reference adaptive control based on hyperstability theory [17] for compensating the hysteresis effect of the PZT actuator in the moving stage system to achieve the precise trajectory tracking objective. Employing the Bouc–Wen model [18] for the PZT hysteresis, some experimental results are given to validate the output-tracking performance of the proposed controller. In addition to numerical simulations, the proposed adaptive control algorithm was applied to an experimental moving stage driven by a PZT actuator and the experimental results indicate the trajectory tracking performance can be improved.

2. Piezo-positioning system

In this paper, a moving stage driven by a PZT actuator is constructed as shown in Fig. 1. One end of the PZT actuator is fixed to the wall and the other end is connected to the moving stage sliding on the horizontal surface. If the frictional force is very small compared to the generating force of the PZT actuator, the physical model of the moving stage system can be depicted in Fig. 2. Applying an input voltage to the PZT actuator, an elongation is produced and then results in a force $F_h$ acting on the imaginary wall. Therefore, based on the Bouc–Wen model [18], the dynamical equation is represented in the following form:

$$m\ddot{x}_1 + b\dot{x}_1 + kx_1 = F_h = k(d_eu - h) \quad (1)$$

where $m$ is the equivalent mass of the PZT actuator and the moving stage, $b$ is the equivalent damping coefficient, $k$ is the equivalent spring coefficient, $u$ is the input voltage, which is applied to the PZT actuator to drive the stage, $x_1$ is the displacement of the stage, $d_e$ is the effective piezoelectric coefficient of the PZT actuator, and $h$ is a variable used for describing the hysteresis effect, respectively.

In this paper, the Bouc–Wen model is used to describe the hysteresis behaviour in the piezoelectric actuator. For the Bouc–Wen model, when the material is uniform elastic, the state variable $h$ forms the hysteresis nonlinear dynamics and is governed by the following equation.

$$\dot{h} = \alpha d_e u - \beta \dot{u} |h| - \gamma |\dot{u}| h \quad (2)$$

where $\alpha$, $\beta$, and $\gamma$ are the parameters adjusting the shapes of the hysteresis loop. Therefore, the block diagram of the moving stage system can be illustrated in Fig. 3.

The system parameters of the moving stage system can be identified using MATLAB identification tool (ARX model) and Simulink software. Generating an 1 kHz unit voltage randomly shown in Fig. 4 from MATLAB software to a voltage amplifier, this input voltage is amplified 10 times to be 0–50 V and then it is sent to the PZT actuator to drive the stage. The displacement response of the stage is measured using a capacitance-type gap sensor having a resolution of 10 nm and is shown in Fig. 5. The system parameters are thus obtained through the system identification of ARX model and listed
there exist positive constants below.

\[ m = 0.28 \text{ kg}, \quad b = 1302.28 \text{ Ns/m}^{-1}, \quad k = 53, 452 \text{ N/m}, \]
\[ d_x = 0.1027 \text{ nm/V}, \quad \alpha = 0.5136, \quad \beta = 0.124 \]
\[ \text{ and } \gamma = -0.073. \]

In order to confirm the validity of system parameters, a sinusoidal input with amplitude =2V and frequency = 1Hz is selected and amplified to actuate the PZT actuator. From the hysteresis loops shown in Fig. 6, it can be seen that experimental hysteresis loops matches the ones of the computer simulation closely.

3. Model reference adaptive control

In this section, a model reference adaptive control will be developed for compensating the hysteresis effect of the PZT actuator to achieve the precise output-tracking aim of the moving stage. Consider an uncertain system with nonlinear input described in the following form:

\[ \dot{x} = Ax(t) + BF(u(t)) + d(x, t) \]
\[ y = Cx(t) \]

where \( x(t) \in \mathbb{R}^n \) is the system state vector, \( u(t) \in \mathbb{R}^m \), \( m \leq n \), is the system control vector, \( y(t) \in \mathbb{R}^p \) is the system output vector, \( A, B, \) and \( C \) are system parameter matrices of appropriate dimensions, \( F(u(t)) = \begin{bmatrix} f_1(u) & \cdots & f_m(u) \end{bmatrix}^T \in \mathbb{R}^m \) is a continuous nonlinear function vector and \( F(0) = 0 \), and \( d(x, t) \in \mathbb{R}^n \) represents system unmodelled errors and external disturbances. In this paper, referring to the definition of a sector bounded function in Ref. [19], a so-called sector-like bounded function is defined as follows:

**Definition 1.** [20] A continuous function \( f_i(u(t)) \) with \( f_i(0) = 0 \) is said to belong to the sector-like bounded \( [c_{i1}, c_{i2}] \) by \( u_i \), if there exist two positive constants \( c_{i1} \) and \( c_{i2} \) such that \( c_{i1} \leq f_i(u_i)/u_i \leq c_{i2} \) for \( u_i \neq 0 \) and \( u = \left[ u_1, u_2, \ldots, u_m \right]^T \).

**Assumption 1.** Nonlinear input functions \( f_i(u(t)), i = 1, \ldots, m, \) are sector-like bounded by \( u_i, \) \( i = 1, \ldots, m, \) respectively. It yields that there exist positive constants \( c_{i1}, i = 1, \ldots, m, \) and \( c_{i2}, i = 1, \ldots, m, \) such that the following conditions are satisfied.

\[ c_{i1} \leq \frac{f_i(u_i)}{u_i} \leq c_{i2}, \quad i = 1, \ldots, m. \]

**Assumption 2.** Uncertain system disturbance vector \( d(x, t) \) is norm bounded. It means that there exists one positive time function \( \lambda(t) \) such that \( ||d(x, t)|| \leq \lambda(t) \).

**Assumption 3.** Input parameter matrix \( B \) has full rank, i.e. \( \text{Rank}(B) = m \).

From **Assumption 1**, it straightly gives the following results:

\[ c_{i1}u_i^2 \leq u_i f_i(u) \leq c_{i2}u_i^2, \quad i = 1, \ldots, m. \]

Then, we have

\[ c_{i1}u_i^2 + c_{i2}u_i^2 + \cdots + c_{m1}u_m^2 \leq u_i f_i(u) + u_2 f_2(u) + \cdots + u_m f_m(u) \]
\[ \leq c_{i1}u_i^2 + c_{i2}u_i^2 + \cdots + c_{m2}u_m^2. \]

It yields that from the above inequality

\[ c_{i1}u_i^2 \leq u_i F(u) \leq c_{i2}u_i^2 \]

where \( c_1 \) and \( c_2 \) are two positive constants with \( c_1 = \min\{c_{i1}; i = 1, \ldots, m\} \) and \( c_2 = \min\{c_{i2}; i = 1, \ldots, m\} \). From **Assumption 1**, it also yields that

\[ c_{i1}u_i^2 \leq F_i^2(u) F(u) \leq c_{i2}u_i^2 u \text{ or } c_1 ||u|| \leq ||F(u)|| \leq c_2 ||u|| \]

The control objective is to find an appropriate control input \( u(t) \) such that the system output vector \( y(t) \) can asymptotically follow the desired output vector \( y_m(t) \) and all the signals in the controlled system are bounded in the presence of system parameter variation, unmodelling error, and external disturbance uncertainties. Here, a stable reference system is given by the following dynamic equation.

\[ \dot{x}_m(t) = A_m x_m(t) + B_m r(t) \]
\[ y_m(t) = C_m x_m(t) \]

where \( x_m(t) \in \mathbb{R}^n \) is the reference model state vector, \( r(t) \in \mathbb{R}^m \) is the reference model input vector, which is piecewise continuous and bounded, \( y_m(t) \in \mathbb{R}^p \) is the reference model output vector, \( A_m \) and \( B_m \) are matrices of appropriate dimension.

To achieve the control objective of output-tracking, in this section we use the hyperstability theory to design a robust adaptive control scheme for the uncertain nonlinear system expressed in Eqs. (3) and (4). Define a state error vector as

\[ e(t) = x(t) - x_m(t). \]

It yields that the dynamics of the state error vector can be represented as

\[ \dot{e} = A_m e + (A - A_m) x + B F(u) + d - B_m r = A_m e - B_1 \omega \]

where \( B_1 \in \mathbb{R}^{n \times n} \) is a designed nonsingular constant matrix and

\[ \omega = -B_1^{-1}[(A - A_m) x + F_1(u) + d - B_m r] \]
\[ F_1(u) = B F(u). \]

We then define a linear combination of state error vector as

\[ R = H e \]

where \( H \in \mathbb{R}^{n \times n} \) is also a designed constant matrix.

Hence, it follows that a linear time invariant system with output \( E \) is given by the following dynamic equation.

\[ \dot{e} = A_m e - B_1 \omega \]
\[ E = H e \]

**Lemma 1.** [21] Let \( G(s) \) be a matrix of rational function such that \( G(\infty) = 0 \) and \( G(s) \) has poles only in \( \text{Re} \{s\} < -\mu, \mu > 0 \). Let \( (H, A, B) \) be a minimal realization of \( G(s) \). Then, \( G(s) \) is strictly positive real if and only if there exist a symmetric, positive definite matrix \( P \) and a matrix \( L \) such that

\[ A^T P + PA = -L L^T - 2 \mu P = -Q \]
\[ PB = H^T \]
of measurement system and environmental noise. Through both numerical and experimental examinations, the proposed adaptive control algorithm can be effectively applied to the positioning devices using PZT actuators to obtain ultraprecision tracking performance.

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References