Adaptive control for a class of chaotic systems with nonlinear inputs and disturbances

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Accepted 29 June 2006

Abstract

In this paper, an adaptive controller design method based on hyperstability theory is proposed for a class of unified chaotic systems with so-called sector-bounded inputs and disturbances. Under the norm-bounded assumption of disturbances, for the controlled system, system states can be regulated to zero levels asymptotically in the presence of this class of disturbances. To show the validity and feasibility of the proposed adaptive controller, some simulations on controlling different chaotic systems with disturbances, are made and investigated.

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1. Introduction

Chaotic systems exhibit unpredictable and irregular dynamics and have been found in many engineering systems, such as lasers [1], Colpitts oscillators [2], nonlinear circuits [3], communication [4], and so on. Since the chaos control problem was first considered by Ott et al. in 1990 [5], it has been investigated extensively by many researchers. Recently, many valuable control methods have been developed to control chaotic systems, such as the sliding mode control method [6,7], adaptive control method [8–10], backstepping method [11], observer-based method [12], and the time-delay feedback control method [13,14]. Recalling all the proposed control methods, we found that the common characteristic of chaos control methods is that the control input of chaotic systems is linear in nature. Owing to physical limitations, there usually exist nonlinearities in the plant actuators of control systems. The presence of nonlinearities in control input may cause serious influence upon system performance. Besides, the control input nonlinearity may result in unpredictable results in chaotic systems. Since the chaotic system is very sensitive to any system parameters, the nonlinear effect in the control input cannot be ignored in both control design and realization for chaotic systems. Considering these points, in this paper we will investigate the control problem of a class of unified chaotic systems with nonlinear inputs.

In this paper, the control objective is to regulate the system states of chaotic systems to reach zeros, asymptotically. In order to achieve the control goal, an adaptive control method based on hyperstability theory [15,16] is developed for chaotic systems with sector-bounded nonlinear inputs, which are subjected to system parameter variations and disturbances. Under the assumptions of sector-bounded nonlinear inputs and norm-bounded disturbances, the proposed

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doi:10.1016/j.chaos.2006.06.090
control scheme has been analyzed theoretically that it has robust asymptotic stability with respect to system uncertainties of parameters, modeling error, and external disturbances. Finally, some numerical simulations are given to validate the proposed adaptive regulating control approach.

Throughout this paper, it is noted that, \(||V||\) represents the Euclidean norm of vector \(V\) and \(\lambda_{\text{min}}(M)\) denotes the minimum eigenvalue of matrix \(M\). This paper is organized as follows. In Section 2 we give a description for a class of chaotic systems. Section 3 introduces the design method of adaptive control for chaotic systems and also presents the stability proof of the proposed adaptive control scheme. Some numerical simulations are provided to illustrate the validity of the proposed control method in Section 4. Finally, a conclusion ends the paper in Section 5.

### 2. Description of the system

Consider the following chaotic system described by

\[
\dot{X} = AX + f(X) + d(X,t),
\]

where \(X(t) \in \mathbb{R}^n\) is a \(n\)-dimensional state vector of the system, \(A \in \mathbb{R}^{n \times n}\) is the matrix of the system parameter, \(f(X): \mathbb{R}^n \rightarrow \mathbb{R}^n\) is the nonlinear part of the system, and \(d(X,t): \mathbb{R}^n \times \mathbb{R}_+ \rightarrow \mathbb{R}^n\) is a disturbance vector. As we know, many chaotic systems investigated, are in a form (1) without disturbance, such as a class of unified chaotic systems in the work of Lu et al. [17], and the Arneodo chaotic system. The unified chaotic system is as follows:

\[
\begin{align*}
\dot{x}_1 &= (25z + 10)(x_2 - x_1), \\
\dot{x}_2 &= (28 - 35z)x_1 - x_1x_3 + (29z - 1)x_2, \\
\dot{x}_3 &= x_1x_2 - \frac{8 + z}{3}x_3,
\end{align*}
\]

where \(z \in [0,1]\). When \(z \in [0,0.8)\), it is the Lorenz chaotic system, \(z = 0.8\) is the Lu chaotic system, and \(z \in (0.8,1]\) is the Chen’s chaotic system. The Arneodo chaotic system is as follows:

\[
\begin{align*}
\dot{x}_1 &= x_2, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= 5.5x_1 - 3.5x_2 - x_3 - x_1^3.
\end{align*}
\]

In this paper, we will investigate the control problem of system (1) with an additional feedback force; and the corresponding controller is designed so that it can render the closed-loop system asymptotically stable under disturbances. Owing to physical limitations, there usually exist nonlinearities in the plant actuators of a control system. The nonlinear effect in control input may cause serious unfavorable influence on chaotic system performance or stability. In view of this reason, a sector-bounded nonlinear input will be considered. In this paper, the control is assumed to be in a form of nonlinear input function vector \(\phi(u)\). Thus the controlled chaotic system is represented by

\[
\dot{X} = AX + f(X) + d(X,t) + \phi(u),
\]

where \(\phi(u) = [\phi_1(u_1) \phi_2(u_2) \cdots \phi_n(u_n)]^T \in \mathbb{R}^n\) is a continuous nonlinear function vector with \(\phi(0) = 0\) and \(u = [u_1 u_2 \cdots u_n]^T \in \mathbb{R}^n\).

The following assumptions specify the class of chaotic systems considered in this paper.

**Assumption 1.** As shown in Fig. 1, nonlinear input functions \(\phi_i(u_i), i = 1,\ldots,n\), are sector bounded by \(u_i, i = 1,\ldots,n\), respectively. It yields the result that there exist positive constants \(c_{i1}, i = 1,\ldots,n\), and \(c_{i2}, i = 1,\ldots,n\), such that the following conditions are satisfied. \(c_{i1} \leq \frac{\phi_i(u_i)}{u_i} \leq c_{i2}\), for \(i = 1,\ldots,n\).

**Assumption 2.** Uncertain disturbance vector \(d(X,t)\) is norm-bounded. It means that there exists a positive time function \(\kappa(t)\), such that \(||d(X,t)|| \leq \kappa(t)\).

### 3. Adaptive control design

From Assumption 1, it can be straightforwardly obtained that

\[
c_{i1}u_i^2(t) \leq u_i(t)\phi_i(u_i(t)) \leq c_{i2}u_i^2(t), i = 1,\ldots,n.
\]
Then, we have
\[c_1 u^2_1(t) + \cdots + c_n u^2_n(t) \leq u_1(t)\phi_1(u_1(t)) + \cdots + u_n(t)\phi_n(u_n(t)) \leq c_2 u^2_1(t) + \cdots + c_n u^2_n(t).\]

Here, there exist two positive constants: \(c_1 = \min\{c_{i1}|i = 1, \ldots, n\}\) and \(c_2 = \max\{c_{i2}|i = 1, \ldots, n\}\). From the above inequality, it yields that
\[c_1 u^T(t)u(t) \leq u^T(t)\phi(u(t)) \leq c_2 u^T(t)u(t).\] (3)

In this paper, we will consider controlling the chaotic system (2) to reach equilibrium 0. It is easy to apply our results to controlling the system’s other fixed points or tracking problems. To achieve the control objective, an adaptive control design based on hyperstability theory will be given in this paper. For designing the controller, the controlled chaotic system (2) is rewritten as
\[
\begin{aligned}
\dot{X} &= A_1X + (A - A_1)X + f(X, t) + \phi(u) = A_1X - B_1\Omega, \\
\Omega &= -B_1^{-1}[(A - A_1)X + f(X) + d(X, t) + \phi(u)],
\end{aligned}
\] (4)

where \(A_1 \in \mathbb{R}^{n \times n}\) is a design Hurwitz matrix, and \(B_1 \in \mathbb{R}^{n \times n}\) is chosen to be a nonsingular constant matrix. Then, a linear combination of system state is defined as
\[Y = CX,\]

where \(C \in \mathbb{R}^{m \times n}\) is also a designed constant matrix. Hence, it follows that a linear time invariant system with output \(Y\) is given by the following dynamic equations:
\[
\begin{aligned}
\dot{X} &= A_1X - B_1\Omega, \\
Y &= CX.
\end{aligned}
\] (6) (7)

According to hyperstability theory, the linear system expressed in Eqs. (6) and (7) is an asymptotically hyperstable system if it satisfies two following conditions:

1. The transfer function
\[G(s) = C(sI - A_1)^{-1}B_1,\]

must be strictly positive real (SPR);

2. The so-called passivity condition (Popov integral inequality)
\[
\int_0^t Y^T(\tau)\Omega(\tau)d\tau \geq -r_0^2\]

is true for all \(t \geq 0\),

where \(r_0\) is an arbitrary finite constant. Then \(G(s)\) is strictly positive real if, and only if, there exists a symmetric, positive definite matrix \(P\) such that

\[P > 0.\]
In this case, matrices $A_1$, $B_1$, and $C$, and all initial values are the same as those used in the previous case. Adaptation gain $\lambda = 0.5$ is applied on the adaptation law. All the behaviors of the proposed adaptive control scheme are illustrated in Figs. 8–13. It can be seen that the system states are regulated to zeros asymptotically even when the nonlinear input chaotic systems are undergoing system parameter abrupt variations and disturbances with modeling errors.

5. Conclusions

In practical control systems, owing to actuator physical limitations, the nonlinearity effect in control input cannot be ignored in the chaotic systems. In this paper, the control problem of chaotic systems with so-called sector-bounded nonlinear input and disturbances is investigated via the adaptive control method. The adaptive controller based on hyperstability theory is designed. Under the norm-bounded assumption of disturbances, the adaptive controller is constructed without the knowledge of norm-bounded value by using an adaptation law. Simulation results show that the proposed adaptive controller can regulate the states of the unified chaotic system to zeros asymptotically, even under the chaotic system, with respect to system parameter variations, modeling errors, and external disturbances.

References


