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What is This?
A Mixed Robust/Optimal Active Vibration Control for Uncertain Flexible Structural Systems with Nonlinear Actuators Using Genetic Algorithm

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Abstract: In this article, a mixed robust/optimal control approach is proposed to treat the active vibration control (or active vibration suppression) problems of flexible structural systems under the effects of mode truncation, linear time-varying parameter perturbations and nonlinear actuators. A new robust stability condition is derived for the flexible structural system which is controlled by an observer-based controller and is subject to mode truncation, nonlinear actuators and linear structured time-varying parameter perturbations simultaneously. Based on the robust stability constraint and the minimization of a defined $H_2$ performance, a hybrid Taguchi-genetic algorithm (HTGA) is employed to find the optimal state feedback gain matrix and observer gain matrix for uncertain flexible structural systems. A design example of the optimal observer-based controller for a simply supported beam is given to demonstrate the combined application of the presented sufficient condition and the HTGA.

Keywords: Active vibration control, parameter perturbations, nonlinear actuators, genetic algorithm

NOMENCLATURE

$I$ The identity matrix.
$\mathbf{I}_n$ The $n \times n$ identity matrix.
$|W|$ The modulus matrix of the matrix $W$.
$W_1 \leq W_2$ $W_{ij,1} \leq W_{ij,2}$, for all i and j; and $W_1, W_2 \in R^{m \times n}$.
$r[W]$ The spectral radius of the matrix $W$.
$\triangleq$ The symbol indicates “defined as”.
$\|w(t)\|$ The symbol denotes a vector by taking $L_1^2$ norm on every element of the vector $w(t) = [w_1(t), w_2(t), ..., w_n(t)]^T$.
$L_p^n[0, \infty)$ The symbol denotes the set of all n-tuples $f(t) = [f_1(t), f_2(t), f_3(t), ..., f_n(t)]^T$, where $f_i(t) \in L_1^n[0, \infty)$, for $i = 1, 2, ..., n$ and $p = 2, \infty$. 

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1. INTRODUCTION

Flexible structural systems are often described by distributed-parameter models and are essentially infinite dimensional. Thus, it is impractical or impossible to implement infinite-dimensional feedback controllers based on complete models of a flexible structural system. Hence, instead of using the original infinite-dimensional distributed parameter model, many researchers use a finite-dimensional model to approximate the original infinite-dimensional model when designing the vibration controller (see, for example, Balas, 1978a,b, 1982; Balas et al., 1988; Lin et al., 1990; Peres et al., 1992; Khot and Heise, 1994; Seto and Mitsuta, 1994; Seto et al., 1995; Khot and Oz, 1997; Chou et al., 1998a; Chen et al., 2000; Zheng et al., 2002; and references therein). Since higher modes have little influence on the motion, they are usually treated as unmodelled dynamics (i.e., they are omitted by the truncation of the original infinite-dimensional model to form the applied finite-dimensional model). Moreover, the finite-dimensional model is divided into two parts: Controlled and residual. The controlled part, which is used for designing the vibration controllers, is composed of those critical modes which make large contributions to the elastodynamic response, and the residual part is composed of the remaining modes of the finite-dimensional model. Two main problems are encountered in the control of flexible structural systems. One is that the residual part may lead to control and observation spillover that can destabilize one or more of the poorly damped modes. The other is that the system is often subject to parameter perturbations due to inaccuracies in the calculations of the frequencies and dampings due to approximations in the structural model, material properties, mass, damping, and so forth. These parameter perturbations can degrade the system performance, and sometimes even destabilize the system.

Most researchers (e.g., Balas, 1978a,b, 1982; Balas et al., 1988; Lin et al., 1990; Peres et al., 1992; Khot and Heise, 1994; Seto and Mitsuta, 1994; Seto et al., 1995; Khot and Oz, 1997) have focused on the problem of suppressing spillover to avoid instability, while the problem of parameter perturbations has received less attention. To the authors’ best knowledge, only a few articles (Lin et al., 1990; Khot and Heise, 1994; Khot and Oz, 1997; Chou et al., 1998a; Chen et al., 2000; Zheng et al., 2002) have addressed the problem of stabilization of flexible structural systems under mode truncation and parameter perturbations. Note that the results proposed by Lin et al. (1990), Chou et al. (1998a), Chen et al. (2000) and Zheng et al. (2002) are valid for linear structured time-varying parameter perturbations, whereas the results given by Khot and Heise (1994) and Khot and Oz (1997) are applicable only to linear structured time-invariant parameter perturbations. It is well known that any analysis used for the time-varying case can be applied to the time-invariant case (but not vice versa). That is, the results of Lin et al. (1990), Chou and Chen et al. (1998), Chen et al. (2000), and Zheng et al. (2002) are valid for both the time-invariant case and the time-varying case.

In terms of perturbations, structured (elemental) perturbations are those for which the structural information of the perturbation matrix is utilized and where bounds on the individual elements of the perturbation matrix are known, whereas unstructured perturbations are those for which only a norm bound on the perturbation matrix is known. The norm expression of the perturbation matrix gives conservative results for the stability estimate of the system. Moreover, the norm measure cannot give sufficient information to determine individual parameter deviations. System engineers often require (or give) the range for deviation of important system parameters in designing the system (Miyagi and Tamashita, 1992). The
linear time-varying parameter perturbations considered by Lin et al. (1990) are categorized as unstructured perturbation cases. The current authors have considered linear structured time-varying perturbation cases in a previous work (Chou et al., 1998a), and showed that the results are less conservative than those given by Lin et al. (1990). Therefore, this article also considers the linear structured time-varying perturbation cases.

On the other hand, the actuator dynamics may possess some nonlinearities, and one of the most common of these is saturation. Actuator saturation may induce unstable output responses and/or lead to undesirable oscillations (Hsu and Meyer, 1968; Chou, 1993). Joshi (1989) proposed a controller synthesis procedure for design of a LQG-type compensator which is robust to mode truncation as well as actuator nonlinearities, but did not take into account structured/unstructured time-varying/time-invariant parameter perturbations. In addition, robust stability by itself is often not enough in control design. The optimal tracking performance is also considered in many practical control engineering applications. Hence, optimal active vibration suppression designs are needed for robust stability and performance design for flexible structural systems under mode truncation, linear time-varying parameter uncertainties and actuator nonlinearities. The optimal active vibration suppression design is to find a stabilizing controller that minimizes the $H_2$ performance index (i.e., the integral of the squared error (ISE) or the integral of the time-weighted squared error (ITSE)) subject to some stability robustness inequality constraints. Therefore, the purpose of this article is to apply the hybrid Taguchi-genetic algorithm (HTGA) to solve the optimal active vibration controller design problems of uncertain flexible structural systems. The reason why the HTGA is applied in this article is that Chou (the second author of this article) and his associates have shown that the HTGA can obtain both better and more robust results than those produced by existing improved genetic algorithms reported in the literature (Chou et al., 1998b; Tsai et al., 2004).

This article is organized as follows: The model of the flexible structural system is described in Section 2. A new sufficient condition for robust stability is proposed in Section 3. Section 4 presents the HTGA for design of the mixed robust/optimal active vibration controller and observer for uncertain flexible structural systems. In Section 5, a simply supported flexible beam is employed as a design example to demonstrate the application of integrating the proposed sufficient condition and the HTGA. Finally, Section 7 offers some conclusions.

2. SYSTEM DESCRIPTION

Consider the flexible structural systems described by a generalized wave equation (Balas, 1978b)

$$m(x)u_{tt}(x, t) + 2\xi A u_t(x, t) + Au(x, t) = f(x, t),$$  \hspace{1cm} (1)

which relates the displacement $u(x, t)$ of the equilibrium position of a body $\Omega$ in the $n$-dimensional space to the applied force distribution $f(x, t)$. The operator $A$ is a time-invariant, symmetric, nonnegative differential operator with a square root $A^{1/2}$, and domain $D(A)$ is dense in the Hilbert space $H = L^2(\Omega)$ with the usual inner product and associated norm. The mass density $m(x)$ is a positive function of the location $x$ on the body; the change of vari-
ables $u(x, t) \rightarrow u(x, t)/m(x)^{1/2}$ eliminates $m(x)$ without changing the properties of equation (1) and, henceforth, we will assume this has been done and take $m(x) = 1$ in this context. The nonnegative number $\xi$ is the damping coefficient of the flexible structural system and depends on the construction materials and methods used. For structures such as spacecraft, $\xi$ may be very small (Balas, 1978b).

We will assume that the spectrum of the operator $A$ contains only isolated eigenvalues $\lambda_k$ with corresponding orthogonal eigenfunctions $\phi_k(x)$ in $D(A)$ such that

$$0 < \lambda_1 < \lambda_2 < \lambda_3 < \ldots \ldots,$$

and

$$A\phi_k(x) = \lambda_k\phi_k(x), \quad A^{1/2}\phi_k(x) = \lambda_k^{1/2}\phi_k(x),$$

with the eigenfunctions forming a basis for $H$; this can be guaranteed by the condition that $A$ has compact resolvent (Balas, 1978b). The eigenfunctions $\phi_k(x)$ are the mode shapes of the flexible structural system, and the mode frequencies are $\omega_k = \lambda_k^{1/2}$.

By applying the standard technique of the expansion theorem with

$$u(x, t) = \sum_{k=1}^{L} u_k(t)\phi_k(x),$$

$$f(x, t) = \sum_{i=1}^{M} f_i(t)\delta(x - x_i),$$

$$y_j(t) = u(q_j, t) \quad (\text{or} \quad y_j(t) = u_i(q_j, t)), \quad j = 1, 2, \ldots, P,$$

where $M$ point actuators (at points $x_i$ on the body) and $P$ point sensors (at various points $q_j$ along the body) are used. Since it is not practical to control all vibrational modes of the flexible structural system, we assume that only some critical modes are considered to be controlled, and thus the distributed parameter system defined in equation (1) can be expressed in the following partitioned finite-dimensional state-space form:

$$\dot{x}_c(t) = A_c x_c(t) + B_c f(t),$$

$$\dot{x}_r(t) = A_r x_r(t) + B_r f(t),$$

$$y(t) = C_c x_c(t) + C_r x_r(t) + y_c(t) + y_r(t),$$

where $x_c(t)$ is the $2N$ dimensional controlled mode state vector and $x_r(t)$ is the $2(L - N)$ dimensional residual mode state vector. The system state vectors contain both displacement and velocity of each mode and are

$$x_c(t) = [u_1(t), u_2(t), \ldots, u_N(t), v_1(t), v_2(t), \ldots, v_N(t)]^T,$$

$$x_r(t) = [u_{N+1}(t), u_{N+2}(t), \ldots, u_L(t), v_{N+1}(t), v_{N+2}(t), \ldots, v_L(t)]^T.$$
in which \( u_k(t) \) represents the modal amplitude of the \( k \)-th mode in normal modal coordinates. The reduced modal velocity \( v_k(t) = u_k(t)/\omega_k \), where \( \omega_k \) is the modal frequency of the \( k \)-th mode. \( y(t) = [y_1(t), y_2(t), ..., y_P(t)]^T \) is the \( P \) dimensional sensor output vector and \( f(t) = [f_1(t), f_2(t), ..., f_M(t)]^T \) is the \( M \) dimensional control force vector. The system matrices are

\[
A_{c,r} = \begin{bmatrix}
0 & \Lambda_{c,r}^T \\
-\Lambda_{c,r}^T & -2\zeta \Lambda_{c,r}^T
\end{bmatrix}, \quad B_{c,r} = \begin{bmatrix}
0 \\
\Lambda_{c,r}^{-1} \bar{B}_{c,r}
\end{bmatrix}, \quad C_{c,r} = \begin{bmatrix}
\bar{C}_{c,r} & 0
\end{bmatrix},
\]

in which the subscripts \( c \) and \( r \) refer to the controlled and residual modes, respectively. \( A_c \) is of dimension \( 2N \times 2N \), \( A_r \) is \( 2(L - N) \times 2(L - N) \), \( B_c \) is \( 2N \times M \), \( B_r \) is \( 2(L - N) \times M \), \( C_c \) is \( P \times 2N \) and \( C_r \) is \( P \times 2(L - N) \). \( \Lambda_{c,r}^T = \text{diag}[\omega_1, \omega_2, ..., \omega_N] \) and \( \Lambda_{c,r}^T = \text{diag}[\omega_{N+1}, \omega_{N+2}, ..., \omega_L] \) with the modal frequencies \( \omega_k \) arranged in increasing order (i.e., \( \omega_k \leq \omega_{k+1} \)). \( \bar{B}_c \) and \( \bar{B}_r \) are actuator influence matrices; \( \bar{C}_c \) and \( \bar{C}_r \) are sensor influence matrices. Note that theoretically \( L \to \infty \). In practice, however, \( L \) may be large but is finite.

That is, the linear finite-dimensional model (5) is only an approximation to the original infinite-dimensional flexible structural system (1).

The modes of the controlled and residual parts can be separated from each other as required by standard mode-reduction techniques (Balas, 1982; Lin et al., 1990), which allows for an additional freedom in the choice of the two parts. Without loss of generality, an assumption is first made with respect to the controlled dynamics: The matrix pair \( (A_c, C_c) \) is completely observable and the pair \( (A_r, B_r) \) is completely controllable. It should be noted that if there are finite unstable poles in the uncontrolled flexible structural systems, we can arrange \( A_c \) so that it contains those nominal unstable poles and the critical modes to be controlled, with \( A_r \) containing the remaining modes.

### 3. ROBUST STABILITY ANALYSIS

Let the matrices \( \Delta A_c(t), \Delta B_c(t) \) and \( \Delta C_c(t) \) denote, respectively, the linear structured time-varying parameter perturbations of the nominal matrices \( A_c, B_c \) and \( C_c \) in the controlled dynamics due to the variations of the natural frequencies and damping ratios due to data errors, changes in operating points, changes in environmental conditions, aging, and any other modeling inaccuracies or changes in the system’s behaviour, properties or environment. In this case, rather than using equations (5a), (5b), and (5c), the actual finite-dimensional system model of the flexible structural system take the form:

\[
\dot{x}_r(t) = A_r x_r(t) + \Delta A_r(t)x_r(t) + B_r N(f(t)) + \Delta B_r(t)N(f(t)),
\]

\[
\dot{x}_c(t) = A_c x_c(t) + B_c N(f(t)),
\]

\[
y(t) = C_c x_c(t) + \Delta C_c(t)x_c(t) + C_r x_r(t).
\]

The parameter perturbation matrices \( \Delta A_r(t), \Delta B_r(t) \) and \( \Delta C_c(t) \) are not known, but their values are constrained to lie within known compact bounding sets. We assume that these perturbation matrices are bounded by the following inequalities:
\[ |\Delta A_i(t)| \leq Q_1, \quad |\Delta B_i(t)| \leq Q_2 \quad \text{and} \quad |\Delta C_i(t)| \leq Q_3, \]

in which \( Q_1, Q_2 \) and \( Q_3 \) are known non-negative constant matrices and represent the structured information available.

**Remark 1.** The controlled model (5a) with \( y_c(t) = C_c x_c(t) \) is used for the design of the control system. The residual (uncontrolled) model (5b) with \( y_r(t) = C_r x_r(t) \) is, of course, present in the system but is neglected in the design of the finite-dimensional controller, and so can be considered as an additive perturbation. Therefore, it is reasonable to assume that no parameter perturbations need to be included for the residual dynamics.

In equations (7) and (8), \( N(f(t)) = [N_1(f_1(t)), N_2(f_2(t)), \ldots, N_M(f_M(t))]^T \) is the \( M \) dimensional nonlinear control force input to the plant, and \( f(t) = [f_1(t), f_2(t), \ldots, f_M(t)]^T \) is the control force input to the actuator. The nonlinear function \( N_i(f_i(t)) \) \((i = 1, 2, \ldots, M)\) which is considered as a model of the actuator nonlinearity must satisfy the twin assumptions that \( N_i(0) = 0 \), and that the nonlinear function \( N_i(f_i(t)) \) is inside the operational sector \([\alpha_i, \beta_i] \), which means that the graph of \( N_i(f_i(t)) \) versus \( f_i(t) \) lies between two straight lines passing through the origin with slopes \( \alpha_i \) and \( \beta_i \), respectively, where \( \alpha_i \) and \( \beta_i \) are real numbers, and \( \beta_i \geq \alpha_i \).

Under these conditions, the following equation (12) is satisfied (Hsu and Meyer, 1968; Chou, 1992, 1993)

\[ |N(f(t)) - U f(t)| = |\Delta S(t) f(t)| \leq D |f(t)|, \]

where

\[
U = \text{diag} \left[ \frac{(a_1 + \beta_1)}{2}, \frac{(a_2 + \beta_2)}{2}, \ldots, \frac{(a_M + \beta_M)}{2} \right], \\
D = \text{diag} \left[ \frac{(\beta_1 - a_1)}{2}, \frac{(\beta_2 - a_2)}{2}, \ldots, \frac{(\beta_M - a_M)}{2} \right],
\]

and

\[
\Delta S(t) = \text{diag} \{ \Delta s_1(t), \Delta s_2(t), \ldots, \Delta s_M(t) \}.
\]

If the actuator is linear, i.e., \( N(f(t)) = f(t) \), which in turn implies \( \alpha_i = \beta_i = 1 \), then equations (7) and (8) can be rewritten as:

\[
\begin{align*}
\dot{x}_c(t) &= A_c x_c(t) + \Delta A_c(t) x_c(t) + B_c f(t) + \Delta B_c(t) f(t), \\
\dot{x}_r(t) &= A_r x_r(t) + B_r f(t).
\end{align*}
\]

Now consider an observer-based active vibration controller, which is designed on the basis of the controlled part, of the form
where
\[
\tilde{Q} = [H_{cr} \tilde{G}_r | H_{cr}] + [H_{cr} \tilde{G}_r | L_1 + L_2,
\]
in which
\[
L_1 = \left[ \begin{array}{cc} |B_c| D |F_c| & |B_c| D |F_c| \end{array} \right],
\]
and
\[
L_2 = \left[ \begin{array}{ccc} Q_1 + Q_2 U |F_c| + Q_2 D |F_c| + |B_c| D |F_c| & Q_2 U |F_c| + Q_2 D |F_c| + |B_c| D |F_c| \\ Q_1 + Q_2 U |F_c| + Q_2 D |F_c| + |K_c| Q_1 & Q_2 U |F_c| + Q_2 D |F_c| \end{array} \right].
\]

Since \( A_r \) is an asymptotically stable matrix, from (A3), (A4), (11) and (16h), we can see that the asymptotic stability of \( z_r(t) \) implies that \( x_r(t) \) is asymptotically stable. Therefore, from (A4), (A5), (A6) and Lemma 2, it follows that if
\[
r \left[ G_c |H_{cr} \tilde{G}_r | H_{cr} + G_c |H_{cr} \tilde{G}_r | L_1 + G_c L_2 \right] < 1,
\]
then system (16) will be asymptotically stable. That is, condition (18) ensures asymptotic stability for the solution to equations (16a) and (16c).

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